‘Nature geometrizes universally in all her manifestations.’ – H.P. Blavatsky

Humans have always looked upon the beauty, majesty, and ingenuity of the world around them with reverent wonder. The grandeur of the heavens, the regular rhythms of the sun and moon, the myriads of living creatures in all their splendid diversity, the magnificence of a rose flower, a butterfly’s wings or a snow crystal – the idea that all
this might be the product of a mindless accident strikes many people as far-fetched. The ancient Greeks aptly called the universe kosmos, a word denoting order and harmony. This underlying order is reflected in many intriguing patterns in nature.

1. Phi and Fibonacci

Consider the line ABC in the following diagram:

![Fig. 1.1](image)

Point B divides the line in such a way that the ratio between the longer segment (AB) and the shorter segment (BC) is the same as that between the whole line (AC) and the longer segment (AB). This proportion is known by various names: the golden section, the golden mean, the golden ratio, the extreme and mean ratio, the divine proportion, or phi (φ). If the distance AB equals 1 unit, then BC = 0.6180339887... and AC = 1.6180339887... . The second of these two numbers is the golden section, or phi (sometimes this name is also given to the first number). Many designs in nature are related to the golden section, and it has been widely used in sacred architecture and artwork throughout the ages.

The golden section is an irrational, or transcendental, number, meaning that it never repeats and never ends. It is unique in that its square is produced by adding the number 1 (φ² = φ+1), and its reciprocal by subtracting the number 1 (1/φ = φ-1). (Phi equals (√5+1)/2, the positive root of x² = x+1.) The golden section is the only irrational number that approaches more closely to rationality the higher the power to which it is raised. For instance, φ³₀₀₀ = 1.0000000000 x 10⁵₀₀.

The golden section is part of an endless series of numbers in which any number multiplied by 1.618 gives the next higher number, and any number multiplied by 0.618 gives the next lower number:
Like phi itself, this series of numbers has many interesting properties:
- each number is equal to the sum of the two preceding numbers;
- the square of any number is equal to the product of any two numbers at equal
distances to the left and right (e.g. $1.618^2 = 0.618 \times 4.236$);
- the reciprocal of any number to the left of 1.000 is equal to the number the same
distance to the right of 1.000 (e.g. $1/0.382 = 2.618$).
(To obtain perfectly exact results, the numbers would have to be extended to an
infinite number of decimals.)

The large rectangle below is a golden rectangle, meaning that its sides are in the
proportion $1.000:1.618$. If a square is removed from this rectangle, the remaining
rectangle is also a golden rectangle. Continuing this process produces a series of
nested golden rectangles. Connecting the successive points where the ‘whirling
squares’ divide the sides of the rectangles in golden ratios produces a logarithmic
spiral, which is found in many natural forms (see next section). A similar spiral can
be generated from a golden triangle (an isosceles triangle whose sides are in the
golden ratio), by repeatedly bisecting one of the angles to generate a smaller golden
deriangle.
A number series closely related to the golden section is the Fibonacci sequence: it begins with 0 and 1, and each subsequent number is generated by adding the two preceding numbers:

\[0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987 ...\]

If we take these numbers two at a time, and divide the larger one by the smaller one, the value will oscillate alternately above and below the golden section, while gradually converging on it. If we divide the smaller number by the larger number, the value converges on \(1/\varphi\) (0.618). (Note that in any series of numbers in which we start with any two ascending numbers and add each consecutive pair to produce the next term, the ratio of successive terms will always tend towards \(\varphi\).)

<table>
<thead>
<tr>
<th>(\frac{1}{1})</th>
<th>(\frac{2}{1})</th>
<th>(\frac{3}{2})</th>
<th>(\frac{5}{3})</th>
<th>(\frac{8}{5})</th>
<th>(\frac{13}{8})</th>
<th>(\frac{21}{13})</th>
<th>(\frac{34}{21})</th>
<th>(\frac{55}{34})</th>
<th>(\frac{89}{55})</th>
<th>(144)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>1.5</td>
<td>1.666</td>
<td>1.600</td>
<td>1.625</td>
<td>1.615385</td>
<td>1.619048</td>
<td>1.617647</td>
<td>1.618182</td>
<td>1.61</td>
</tr>
</tbody>
</table>

The Fibonacci sequence has many curious features. For example, the sum of any 10 consecutive Fibonacci numbers is always divisible by 11, being equal in fact to 11 times the seventh number. The sum of all Fibonacci numbers from the first to the nth is equal to the (n+2)th number minus 1. Thus the sum of the first 15 numbers (986) is equal to the 17th number (987) minus 1. The square of any term differs by 1 from the product of the two adjacent terms in the sequence: e.g. \(3^2 = 9, 2 \times 5 = 10; 13^2 = 169, 8 \times 21 = 168\). The last digit of any number in the sequence repeats itself with a periodicity of 60: e.g. the 14th number and the 74th number both end in 7. The last two digits (01, 01, 02, 03, 05, 08, 13, 21, etc.) repeat in the sequence with a periodicity of 300. For any number of last digits from three upwards, the periodicity is 15 times ten to a power that is one less than the number of digits, (e.g. for 7 digits it is \(15 \times 10^6\), or 15 million).

2. Nature’s numbers

Many of nature’s patterns are related to the golden section and the Fibonacci numbers. For instance, the golden spiral is a logarithmic or equiangular spiral – a type of spiral found in unicellular foraminifera, sunflowers, seashells, animal horns and tusks, beaks and claws, whirlpools, hurricanes, and spiral galaxies. An equiangular spiral does not alter its shape as its size increases. Because of this remarkable property (known as self-similarity),* it was known in earlier times as the ‘miraculous spiral’.

*Shapes and forms that look similar under any magnification are known as fractals. Numerous apparently irregular natural phenomena display approximate self-similarity, meaning that similar patterns and details recur at smaller and smaller scales. This applies, for example, to the
branching of lightning, rivers, trees, and human lungs. Other fractal objects include the chambered nautilus, a head of cauliflower, coastlines, clouds, snowflakes, and rocks (a magnified rock can look like an entire mountain).

Fig. 2.1 The Whirlpool Galaxy (M51).

Each increment in the length of the nautilus shell is accompanied by a proportional increase in its radius, so that the nautilus does not need to adjust its balance as it matures.

Fig. 2.2 Each increment in the length of the nautilus shell is accompanied by a proportional increase in its radius, so that the nautilus does not need to adjust its balance as it matures.
Fig. 2.3 A ram's heavy corkscrewed horns keep a stable centre of gravity as they grow.

Fig. 2.4 Owing to the structure of their compound eyes, insects such as moths follow an equiangular spiral when drawn towards a candle flame. Peregrine falcons, which have eyes on either side of their heads, follow a similar spiral path when flying at their prey.

Phi can be found in the proportions of the bodies of humans and other creatures, including birds, flying insects, frogs, fish, and horses. The height of a newborn child is divided by the navel into two equal parts (1:1), whereas in adults, the division of the body's height at the navel yields two parts in the ratio 1:φ, though the navel is usually a little higher in females and a little lower in males. Phi is found in the proportion between the hand and the forearm, and between the upper arm and the hand plus forearm.

Fig. 2.5 Each section of an index finger, from the tip to the base of the wrist, is larger than the preceding one by about 1.618, fitting the Fibonacci numbers 2, 3, 5 and 8.

The DNA molecule measures 34 angstroms long by 21 angstroms wide for each full cycle of the double helix spiral. 34 and 21 are Fibonacci numbers and their ratio
closely approximates phi. DNA has two grooves in its spirals, with a phi ratio between the major groove and the minor groove (roughly 21 angstroms to 13 angstroms).

![DNA Image](image)

**Fig. 2.6**

The Fibonacci numbers and golden section are widely found in the plant kingdom. In nearly all flowers, the number of petals is a Fibonacci number. For instance, lilies have 3 petals, buttercups have 5, many delphiniums have 8, marigolds have 13, asters have 21, and daisies commonly have 13, 21, 34, 55 or 89. Non-Fibonacci numbers don’t occur anything like as often. For instance, very few plants have 4 petals, an exception being the fuchsia. Some plant species are very precise about the number of petals they have, e.g. buttercups, but with others only the average number of petals is a Fibonacci number.

![Shasta Daisy](image)

**Fig. 2.7** Shasta daisy with 21 petals.

Fibonacci numbers are often found in the arrangement of branches, leaves, and seeds (phyllotaxis). If we look at a plant from above, the leaves are not arranged directly above one another, but in a way that optimizes their exposure to sun and rain. The Fibonacci numbers occur when counting both the number of times we go
around the stem from one leaf to the next, and when counting the number of leaves we meet until we encounter one directly above the starting one. The number of turns in each direction and the number of leaves are usually three consecutive Fibonacci numbers.

In the diagram below, to get from the topmost leaf to the last of the 5 leaves of the plant on the left takes 2 anticlockwise turns or 3 clockwise turns. 2, 3 and 5 are three consecutive Fibonacci numbers. For the plant on the right, it takes 3 anticlockwise rotations or 5 clockwise rotations to pass 8 leaves. Again, 3, 5 and 8 are consecutive Fibonacci numbers. An estimated 90% of all plants display this pattern.

<table>
<thead>
<tr>
<th>Fibonacci fraction: leaves/spirals</th>
<th>5/2</th>
<th>5/3</th>
<th>8/3</th>
<th>8/5</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle between consecutive leaves</td>
<td>144°</td>
<td>216°</td>
<td>135°</td>
<td>225°</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Leaf arrangement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Apple, cherry, apricot, oak, cypress, poplar</td>
<td>Holly, pear, spruce, various beans</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Similar arrangements can be found in the scales of a pinecone or the seeds of a sunflower. The florets in the head of a sunflower form two intersecting sets of spirals, one winding clockwise, the other anticlockwise. In some species the number of clockwise spirals is 34, and the number of counterclockwise spirals is 55. Other possibilities are 55 and 89, or 89 and 144. Pineapples have 8 rows of scales sloping to the left, and 13 to the right. Again, these are successive Fibonacci numbers.

![Sunflower head](image)

**Fig. 2.9** Sunflower head.

![Pine cone](image)

**Fig. 2.10** This pine cone has 13 spirals to the right and 8 to the left.

The fact that in the arrangement of seeds on flower heads, the numbers of spirals in each direction are nearly always neighbouring Fibonacci numbers means that each seed is roughly 0.618 of a turn from the last one, i.e. there are roughly 1.618 seeds per turn. This results in optimal packing of the seeds, because no matter how big the seed head gets, the seeds are always equally spaced. Similarly, trees and plants tend to have 0.618 leaves or petals per turn. In terms of degrees this is 0.618034 of 360°, which is 222.5°. If we measure the angle going in the opposite direction around the circle, we get: 360 - 222.5 = 137.5°, which is known as the golden angle. The Fibonacci numbers appear in leaf arrangements and the number of spirals on seedheads because they form the best whole-number approximations to the golden
section. The higher the numbers are on the Fibonacci sequence, the more closely the golden ratio and the golden angle are approached and the more complex the plant.

3. Pentads and hexads

Fig. 3.1 Each successive pair of heavy lines in this pentagram is in the golden ratio.

The golden ratio appears in pentagonal forms of symmetry, notably in the five-pointed star (or pentagram), which was the emblem of the Pythagorean brotherhood. Pentagonal symmetry abounds in living organisms, especially plants and marine animals (e.g. starfishes, jellyfishes, sea urchins). Flowers with 5 petals (or multiples of 5) include all fruit blossoms, water lilies, roses, honeysuckles, carnations, geraniums, primroses, orchids, and passionflowers.

Fig. 3.2 Left: Sea cucumber (cross section). Right: Sand dollar (sea urchin).

Fig. 3.3 Pentagonal diatom.
Fig. 3.4 Apple blossoms have five petals, pentagonal indentations are seen at the bottom of the fruit, and cutting an apple in half reveals a star pattern of seeds.

Fig. 3.5 The human body clearly expresses the pentad’s symmetry in its five senses and five extensions from the torso, each limb ending in five fingers or toes.

While pentagonal patterns abound in living forms, the mineral world favours twofold, threefold, fourfold, and sixfold symmetry. The hexagon is a ‘close-packing’ shape that allows for maximum structural efficiency. It is very common in the realm of molecules and crystals, in which pentagonal forms are almost never found. Steroids, cholesterol, benzene, TNT, vitamins C and D, aspirin, sugar, graphite – all show sixfold symmetry. The most famous hexagonal architecture is built by bees, wasps, and hornets.
Fig. 3.6 Six water molecules form the core of each snow crystal.

Fig. 3.7 Snow crystals (courtesy of Kenneth G. Libbrecht).

Fig. 3.8 Benzene ($\text{C}_6\text{H}_6$).
Fig. 3.9 Honeycomb.

Fig. 3.10 The facets of a fly’s eyes form a close-packed hexagonal arrangement.

Fig. 3.11 A hexagonal lattice, or honeycomb, of convection cells (Bénard cells) in a heated liquid.

The atmosphere of Saturn, one of the four gas giants, circulates around its axis in striped bands. Seen from the poles, the bands generally appear to be circular. But the band nearest the north pole is hexagonal, with sides about 13,800 km long. Unlike other clouds in its atmosphere, it rotates slowly, if at all, relative to the planet. It is one of Saturn’s most enigmatic features.
4. Platonic solids

A regular polyhedron is a three-dimensional shape whose edges are all of equal length, whose faces are all identical and equilateral, and whose corners all touch the surface of a circumscribing sphere. There are only five regular polyhedra: the tetrahedron (4 triangular faces, 4 vertices, 6 edges); the cube/hexahedron (6 square faces, 8 vertices, 12 edges); the octahedron (8 triangular faces, 6 vertices, 12 edges); the dodecahedron (12 pentagonal faces, 20 vertices, 30 edges); and the icosahedron (20 triangular faces, 12 vertices, 30 edges). In each case, the number of faces plus corners equals the number of edges plus 2. These five polyhedra are also known as the platonic or pythagorean solids, and are intimately connected with the golden section. Their striking beauty is derived from the symmetries and equalities in their relations.
Fig. 4.1 The five platonic solids. The ancients considered the tetrahedron to represent the element of fire; the octahedron, air; the icosahedron, water; and the cube, earth. The dodecahedron symbolized the harmony of the entire cosmos.

Fig. 4.2 An icosahedron’s 12 vertices are defined by three perpendicular golden rectangles. With an edge length of one unit, its volume is $5\phi^{5/6}$.

The symmetry of the platonic solids leads to other interesting properties. For instance, the cube and octahedron both have 12 edges, but the numbers of their faces and vertices are interchanged (cube: 6 faces and 8 vertices; octahedron: 8 faces and 6 vertices). Similarly, the dodecahedron and icosahedron both have 30 edges, but the dodecahedron has 12 faces and 20 vertices, while for the icosahedron it is the other way round. This allows one solid to be mapped into its dual or reciprocal solid. If we connect the centres of all the faces of a cube, we obtain an octahedron, and if we connect the centres of the faces of an octahedron, we obtain a cube. The same procedure can be used to map an icosahedron into a dodecahedron, and vice versa. (In the Hindu tradition, the icosahedron represents purusha, the male, spiritual principle, and generates the dodecahedron, representing prakriti, the female, material principle.) The tetrahedron is self-dual – joining the four centres of its faces produces another, inverted tetrahedron.
There are two other ways of passing from dodecahedron to icosahedron, and icosahedron to dodecahedron. If we join interiorly all the vertices of an icosahedron, the lines will intersect at 20 points defining the vertices of a dodecahedron. If we then do the same with the resulting dodecahedron, a smaller icosahedron is generated within it, and so on, ad infinitum. Likewise, if we lengthen the sides of an icosahedron, an enveloping dodecahedron will be formed. Lengthening the sides of the dodecahedron will generate an enveloping icosahedron. Again, this operation can be repeated indefinitely – a fitting symbol of the theosophical teaching of worlds within worlds.

The sum of the angles of the platonic solids is 3600 degrees for the icosahedron, 6480 for the dodecahedron, 1440 for the octahedron, 2160 for the cube, and 720 for the tetrahedron. Each of these numbers is divisible by 9, which means that the sum of the digits is 9 (e.g. 6+4+8=18, 1+8=9). They are also divisible by the canonical numbers 12, 60, 72 and 360. As we shall see, the numerical elements of the five regular polyhedra frequently recur in nature’s cycles.

The platonic solids (especially the tetrahedron, octahedron, and cube) form the basis for the orderly arrangement of atoms in crystals, though the regular dodecahedron and icosahedron are never found.
Fig. 4.4 In salt crystals, the sodium chloride atoms tightly pack along cubic lines of force.

Tetrahedral geometry commonly occurs in organic and inorganic chemistry and in submicroscopic structures. For example, the methane molecule (CH₄) is a tetrahedron, with a carbon atom at its centre and a hydrogen atom at each of its four corners.

Carbon exists in three pure forms. In graphite crystals, the carbon atoms lie in hexagonal sheets, which readily slide off a pencil as we write. In diamond, the hardest substance known, each carbon atom is bonded to four others in a super-strong tetrahedral arrangement. Buckminsterfullerene, the third, highly stable allotrope of carbon, consists of 60 carbon atoms, arranged at the vertices of a truncated icosahedron (i.e. one with its corners cut off).

Fig. 4.5 Graphite; diamond; buckminsterfullerene.

The great majority of viruses are icosahedral, including the polio virus and the 200 kinds of viruses responsible for the common cold. Icosahedral symmetry is believed to allow for the lowest-energy configuration of particles interacting on the surface of a sphere. The five platonic solids are also found in radiolarian skeletons.
Fig. 4.6 The platonic solids have been found living in the sea. The tetrahedron, somewhat rounded as if from internal pressure, is embodied in a protozoan called *Callimitra agnesae*, the cube is *Lithocubus geometricus*, the octahedron *Circoporus octahedrus*, the dodecahedron *Circorrhegma dodecahedrus*, and the icosahedron *Circognia icosahedrus*.

5. Precession and yugas

Due to a slow gyration of the earth’s axis, the spring equinox occurs about 20 minutes earlier each year, when the earth is still about 1/72 of a degree (50 arc-seconds) from the point in its orbit where the previous spring equinox occurred. The vernal equinoctial point therefore moves slowly through the constellations of the zodiac. This key astronomical cycle is known as the precession of the equinoxes. The ancients called it the ‘great year’.

At the average rate of 1/72 of a degree per year, the earth enters a new constellation about every 2160 years, and takes 25,920 years to make a complete circuit of the zodiac. The entry into each new constellation marks the beginning of what is called in theosophy the Messianic cycle, which is said to be marked by the appearance on earth of a spiritual teacher or avatar (lit. a ‘descent’ of divinity); for instance, the commencement of the Piscean age was marked by the appearance of Christ. Interestingly, 2160 is the total number of degrees in the angles of a cube, and if a cube is opened out it forms the shape of a cross, a universal symbol representing the descent of spirit into matter, or the ‘crucifixion’ of spirit in the ‘tomb’ of matter.

The ancient chronological charts of the Brahmans refer to four great cycles or yugas. The krita- or satya-yuga lasts 400 divine years, the treta-yuga 300, the dvapara-yuga 200, and the kali-yuga 100, a ‘divine year’ being equal to 360 earth-years. Each yuga is introduced by a ‘dawn’ and concluded by a ‘twilight’, each equal to one-tenth of the period of the yuga. The total length of the four yugas in earth-years is therefore 1,728,000 for the satya-yuga, 1,296,000 for the treta-yuga, 864,000 for the dvapara-yuga, and 432,000 for the kali-yuga. The lengths of these four yugas are related in the ratio 4:3:2:1, i.e. they are all multiples of 432,000. If the individual digits of each yuga are added together, the result is always 9 (e.g. 1+2+9+6=18, 1+8=9).

The four yugas together make up one maha-yuga, lasting 4,320,000 years, which in theosophy is said to equal half the evolutionary period of a root-race or ‘humanity’, of which we are now in the fifth. The total life-period of earth, or a day of Brahma, is said to last 1000 maha-yugas or 4,320,000,000 years, and is followed by a rest period, or night of Brahma, of the same length, making a total of 8,640,000,000 years. 360 days and nights make up one year of Brahma, equal to 3,110,400,000,000 ordinary years. 100 years of Brahma make up an age of Brahma, this being the total life-period of our universal solar system, equal to
311,040,000,000,000 years. If we take the precessional cycle of 25,920 years and add to it a dawn and a twilight equal to one-tenth of its length, we get 31,104 – the initial digits of a year and age of Brahma.

4320 is twice 2160, the total number of degrees in a cube, the length of the Messianic cycle, and the average time it takes the equinoctial point to pass through one constellation of the zodiac. 432,000 is equal to 4 x 108,000, and 108,000 years is the average length of an astronomical cycle known as the revolution of the line of apsides. The line of apsides is the line joining that point in the earth’s elliptical orbit where it is closest to the sun (perihelion) and that point where it is furthest from the sun (aphelion). This line turns extremely slowly – by an average of 12 arc-seconds a year – from west to east, and therefore rotates through the whole zodiac in 108,000 years.

The sun has a radius of approximately 432,000 miles (432,475) and a diameter of about 864,000 miles, while the moon has a radius of 1080 miles and a diameter of 2160 miles. Silver, the metal associated with the moon, has an atomic weight of 107.9. In addition, 108 is roughly the average distance between the sun and earth in terms of solar diameters, the average distance between the surfaces of the moon and earth in terms of lunar diameters, and the diameter of the sun in terms of earth diameters (actual figures: 107.5, 108.3, and 109.1 respectively). As a result of these remarkable ‘coincidences’, the moon has the same apparent size as the sun, as seen from earth, and almost exactly covers the disk of the sun during a total solar eclipse.
According to conventional astronomy, in each precessional cycle the earth’s north pole slowly traces an approximate circle, with an average radius of about 23.5 degrees (the current inclination of its axis), around the north pole of the ecliptic – a point in the constellation Draco perpendicular to the plane of the earth’s orbit around the sun. (According to theosophy, the earth’s axis does not trace a circle but a spiral around the ecliptic poles, as the tilt of the earth’s axis is said to change by four degrees every precessional cycle.)

The ‘circle’ traced by the earth’s axis around the north ecliptic pole is not smooth but wavy, as the moon’s gravitational pull causes the earth to ‘nod’ about once every 18 years (currently 18.6 years), a movement known as nutation. The circle therefore contains about 1440 waves, since 18 x 1440 = 25,920. Note that the average human heartbeat is equal to 72 beats a minute, and on average we breathe about 18 times a minute. 72 years (= 6+60+6 or 6x12) is said to be the ‘ideal’ lifespan of a human being, during which time the sun moves through one degree of the zodiac in the precessional cycle. A human breathes 72 times in 4 minutes, the time required for the earth to turn 1 degree on its axis. In 24 hours (86,400 seconds) we breathe 18 x 1440 = 25,920 times, equal to the number of years in the precessional cycle.

Each human heartbeat takes about 8 tenths of a second. The time-periods occupied by each of the five cardiac phases, reckoned on the basis of one hour, are as follows: auricular systole, 432 seconds; ventricular systole, 1296 seconds; repose of the entire heart, 1728 seconds; diastole, 3024 seconds; and ventricular diastole, 2160 seconds. There are on average 4320 heartbeats in one hour, 8640 beats in 2 hours, 12,960 beats in 3 hours, 17,280 beats in 4 hours, 21,600 beats in 5 hours, and 25,920 beats in 6 hours.* In these figures we can recognize the digits of the four yugas, the Messianic cycle, and the precessional cycle (whose lengths are all multiples of 12, 60, 72, and 360). Thus, again and again we find correspondences between what takes place in the small (the microcosm) and what takes place in the great (the macrocosm) – as above, so below.