#55. $\dfrac{\sin(A-B)}{\cos A \cos B} = \tan A - \tan B$

$$\frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}$$

$$\frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B}$$

$$\tan A - \tan B = \tan A - \tan B$$

#57. $\sec(A+B) = \dfrac{\cos(A-B)}{\cos^2 A - \sin^2 B}$

$= \dfrac{\cos A\cos B + \sin A\sin B}{\cos^2 A - \sin^2 B}$ $(\cos A\cos B - \sin A\sin B)$

$\dfrac{1}{\cos A\cos B - \sin A\sin B} = \dfrac{(\cos^2 A\cos^2 B - \sin^2 A\sin^2 B)}{(\cos^2 A - \sin^2 B)(\cos A\cos B - \sin A\sin B)}$

$= \dfrac{(\cos^2 A\cos^2 B - \sin^2 A\sin^2 B)(\cos^2 A - \sin^2 B)}{\cos A\cos B - \sin A\sin B}$

$\sin^2 + \cos^2 = 1$

$\sin^2 = 1 - \cos^2$

$-\sin^2 = \cos^2 - 1$

$= \cos^4 A\cos^2 B - \cos^2 A\cdot\cos^2 B\sin^2 B\cdot$?
$-\sin^2 B\cdot\sin^2 A\cos^2 A + \sin^2 A\cdot\sin^4 B$

$= \cos^4 A\cdot\cos^2 B - \cos^2 A\cdot\cos^2 B\cdot\sin^2 B$
$-\sin^2 B\cdot(\sin^2 - \sin^4 A)$

this problem #57 was worked on in class - dead end in class. The problem is correct up to the red line. I do not see where I got the term the question mark points to. See the next page for a correct finish to the problem.

#57. $\sec(A+B) = \dfrac{\cos(A-B)}{\cos^2 A - \sin^2 B}$

$= \dfrac{\cos A\cos B + \sin A\sin B}{\cos^2 A - \sin^2 B}$ $(\cos A\cos B - \sin A\sin B)$

$\dfrac{1}{\cos A\cos B - \sin A\sin B} = \dfrac{(\cos^2 A\cos^2 B - \sin^2 A\sin^2 B)}{(\cos^2 A - \sin^2 B)(\cos A\cos B - \sin A\sin B)}$

$= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\cdot\sin^2 B$

$= \dfrac{\cos^2 A - \cos^2 A\sin^2 B - (\sin^2 B - \cos^2 A\sin^2 B)}{(\cos^2 A - \sin^2 B)(\cos A\cos B - \sin A\sin B)}$

$= \dfrac{\cos^2 A - \cos^2 A\sin^2 B - \sin^2 B + \cos^2 A\sin^2 B}{(\cos^2 A - \sin^2 B)(\cos A\cdot\cos B - \sin A\sin B)}$

$= \dfrac{\cos^2 A - \sin^2 B}{(\cos^2 A - \sin^2 B)(\cos A\cdot\cos B - \sin A\sin B)}$

$= \dfrac{1}{\cos A\cos B - \sin A\sin B} = \dfrac{1}{\cos(A+B)} = \sec(A+B)$

$\sin(A+B)$ $\quad$ $\sin(A+A)$

$\sin A\cos A + \cos A \cdot \sin A$

$$\boxed{\sin 2A = 2\sin A\cos A}$$

$\cos(A+B)$ $\quad$ $\cos(A+A)$

$\cos A\cos A - \sin A \cdot \sin A$

$$\boxed{\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 1 - 2\sin^2 A \\ &= 2\cos^2 A - 1\end{aligned}}$$

$\tan(A+B)$ $\quad$ $\tan(A+A) = \dfrac{\tan A + \tan A}{1 - \tan A \cdot \tan A}$

$$\boxed{\tan 2A = \frac{2\tan A}{1-\tan^2 A}}$$