#57. $\sec(A+B) = \dfrac{\cos(A-B)}{\cos^2 A - \sin^2 B}$

$= \dfrac{\cos A \cos B + \sin A \sin B}{\cos^2 A - \sin^2 B} \cdot \dfrac{(\cos A \cos B - \sin A \sin B)}{(\cos A \cos B - \sin A \sin B)}$

$\dfrac{1}{\cos A \cos B - \sin A \sin B}$

$= \dfrac{(\cos^2 A \cos^2 B - \sin^2 A \sin^2 B)}{(\cos^2 A - \sin^2 B)(\cos A \cos B - \sin A \sin B)}$

$= \dfrac{(\cos^2 A \cos^2 B - \sin^2 A \sin^2 B)(\cos^2 A - \sin^2 B)}{\cos A \cos B - \sin A \sin B}$

$\sin^2 + \cos^2 = 1$

$\sin^2 = 1 - \cos^2$

$-\sin^2 = \cos^2 - 1$

$= \cos^4 A \cos^2 B - \cos^2 A \cdot \cos^2 B \sin^2 B \cdot$
$- \sin^2 B \cdot \sin^2 A \cos^2 A + \sin^2 A \cdot \sin^4 B$?

$= \cos^4 A \cdot \cos^2 B - \cos^2 A \cdot \cos^2 B \cdot \sin^2 B$
$- \sin^2 B \cdot (\sin^2 - \sin^4 A)$

this problem #57 on the 5.2 assignment was worked on in class - dead end in class. The problem is correct up to the red line. I do not see where I got the term the question mark points to. See the next page for a correct finish to the problem.

#57. $\sec(A+B) = \dfrac{\cos(A-B)}{\cos^2 A - \sin^2 B}$

$= \dfrac{\cos A \cos B + \sin A \sin B}{\cos^2 A - \sin^2 B} \cdot \dfrac{(\cos A \cos B - \sin A \sin B)}{(\cos A \cos B - \sin A \sin B)}$

$\dfrac{1}{\cos A \cos B - \sin A \sin B}$

$= \dfrac{(\cos^2 A \cos^2 B - \sin^2 A \sin^2 B)}{(\cos^2 A - \sin^2 B)(\cos A \cos B - \sin A \sin B)}$

$= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\cdot \sin^2 B$

$= \dfrac{\cos^2 A - \cos^2 A \sin^2 B - (\sin^2 B - \cos^2 A \sin^2 B)}{(\cos^2 A - \sin^2 B)(\cos A \cos B - \sin A \sin B)}$

$= \dfrac{\cos^2 A - \cos^2 A \sin^2 B - \sin^2 B + \cos^2 A \sin^2 B}{(\cos^2 A - \sin^2 B)(\cos A \cdot \cos B - \sin A \sin B)}$

$= \dfrac{\cos^2 A - \sin^2 B}{(\cos^2 A - \sin^2 B)(\cos A \cdot \cos B - \sin A \sin B)}$

$= \dfrac{1}{\cos A \cos B - \sin A \sin B} = \dfrac{1}{\cos(A+B)} = \sec(A+B)$

#45. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$=$

$\sin(2\theta + \theta) =$

$\sin 2\theta \cos\theta + \cos 2\theta \sin\theta$,

$2\sin\theta\cos\theta \cdot \cos\theta + (\cos^2\theta - \sin^2\theta) \cdot \sin\theta$

$2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$

$3\sin\theta\cos^2\theta - \sin^3\theta$

$3\sin\theta(1 - \sin^2\theta) - \sin^3\theta$

$3\sin\theta - 3\sin^3\theta - \sin^3\theta$

$3\sin\theta - 4\sin^3\theta = 3\sin\theta - 4\sin^3\theta$

$$\#53. \quad \frac{1-\tan x}{1+\tan x} = \frac{1-\sin 2x}{\cos 2x}$$

$$\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}} =$$

$$\frac{\cos x-\sin x}{\cos x} \cdot \frac{\cos x}{\cos x+\sin x} =$$

$$\frac{\cos x-\sin x}{\cos x+\sin x}\,\frac{(\cos x-\sin x)}{(\cos x-\sin x)} = \frac{\cos^2 x-2\sin x\cos x+\sin^2 x}{\cos^2 x-\sin^2 x}$$

$$= \frac{1-\sin 2x}{\cos^2 x-(1-\cos^2 x)}$$

$$= \frac{1-\sin 2x}{2\cos^2 x-1}$$

$$= \frac{1-\sin 2x}{\cos 2x}$$

$$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1-\cos A}{\sin A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A} \cdot \frac{(1-\cos A)}{(1-\cos A)} \quad \frac{\sin A(1-\cos A)}{\sin^2 A}$$

$$\tan\left(\frac{A}{2}\right) = \frac{\sqrt{\frac{1-\cos A}{2}}}{\sqrt{\frac{1+\cos A}{2}}} = \frac{\sqrt{1-\cos A}}{\sqrt{1+\cos A}} \cdot \frac{\sqrt{1+\cos A}}{\sqrt{1+\cos A}} = \frac{\sqrt{\sin^2 A}}{1+\cos A}$$

$$= \frac{\sin A}{1+\cos A}$$