Department of Mathematics

Wisconsin Lutheran College

EXAM 1

MAT 223

Name _____

September 17, 2007

- You have 50 minutes to complete the exam.
- Partial credit will be given so you must SHOW ALL OF YOUR WORK. Put all of your work and answers in the space provided. Scratch paper is not allowed.
- Place your books, notebooks, etc. on the floor. The only items on your desk should be this exam and pencil/eraser/pen. Calculators are not allowed.

Problem	Points	Points per part	Points Earned
True-False	12	2	
1	10	5	
2	6		
3	6		
4	6		
5	24	6	
6	6		
7	6		
8	6		
9	6		
10	6		
11	6		
TOTAL	100		

Exam 1
MAT 223

I. True-False

T F a. Two vectors are orthogonal if and only if their dot product is 0.
T F b. The dot product operation for vectors is not commutative.
T F c. In $\mathbb{R}^3$ a point and a nonzero vector uniquely determine a plane.
T F d. For any vectors $\mathbf{u}, \mathbf{v}$, $3(\mathbf{u} \times \mathbf{v}) = \mathbf{u} \times (3\mathbf{v})$.
T F e. For any vectors $\mathbf{u}, \mathbf{v}$, $(\mathbf{u} \times \mathbf{u}) \cdot \mathbf{v} = 0$.
T F f. In spherical coordinates the second coordinate is an angle between 0 and $\pi/2$.

II. Definitions

1. Identify the quadric surface having the given equation:

   (a) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$

   (b) $z = \dfrac{y^2}{b^2} + \dfrac{x^2}{a^2}$

III. Applications

2. For the vector $\mathbf{u} = < -2, 1, 3 >$ find a unit vector in the same direction.

3. Given the vectors $\mathbf{u}, \mathbf{v}$ shown below, draw the vectors $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

4. Find the component form of the vector $\mathbf{v}$ having length 3 and making an angle of $\pi/6$ with the positive x-axis.

5. For the vectors $\mathbf{u} = < 1, 1, 1 >$ and $\mathbf{v} = < -2, 1, 1 >$, find

   (a) The cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.

   (b) The direction cosines of $\mathbf{u}$.

(c) The component (scalar projection) of $\mathbf{u}$ in the $\mathbf{v}$ direction.

(d) $\mathbf{u} \times \mathbf{v}$

6. Find symmetric equations for the line through the point $(4, -5, 2)$ and parallel to the vector $< -3, 2, 1 >$.

7. Find parametric equations for the line through the point $(3, 2, 4)$ in the direction of the y-axis.

8. Find the standard equation of the plane that passes through the point $(7, -1, 3)$ and has normal vector $\mathbf{n} = < 5, 2, 3 >$.

9. Describe the surface whose equation is $r = \pi/4$ in cylindrical coordinates.

10. Convert the equation $3x^2 + 2y^2 + z^2 = 1$ to an equation in cylindrical coordinates. (Do NOT simplify!)

11. Show that for ANY vector $\mathbf{v} = < v_1, v_2, v_3 >$, $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$.