Department of Mathematics

Wisconsin Lutheran College

EXAM 2
MAT 223

Name ______________________________

October 24, 2007

- You have 50 minutes to complete the exam.
- Partial credit will be given so you must SHOW ALL OF YOUR WORK. Put all of your work and answers in the space provided. Scratch paper is not allowed.
- Place your books, notebooks, etc. on the floor. The only items on your desk should be this exam and pencil/eraser/pen. Calculators are not allowed.

Problem	Points	Points per part	Points Earned
True-False	10	2	
1	6		
2	6		
3	6		
4	6		
5	24	6	
6	7		
7	7		
8	7		
9	7		
10	7		
11	7		
TOTAL	100		

I. True – False

T F a. For a curve parametrized by arclength, the length of the tangent vector at any point on the curve is 1.

T F b. For a curve parametrized by arclength, curvature is the length of the acceleration vector.

T F c. If f, f_x, f_y are continuous in an open region R of the plane, then $f(x, y)$ is differentiable in R.

T F d. If $f(x, y)$ is differentiable at $(x_{0,}y_0)$ then f is continuous at $(x_{0,}y_0)$.

T F e. A nonzero gradient of f is always normal to the corresponding level curve of f.

II. Applications

1. Compute $D_t(\mathbf{r}(t)\bullet\mathbf{u}(t))$ where $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + 1\mathbf{k}$ and $\mathbf{u}(t) = t^{-2}\mathbf{i} + \sin t\mathbf{j} + \cos t\mathbf{k}$.

2. Find $\mathbf{T}$ for the curve $\mathbf{r}(t) = t^2\mathbf{i} + \mathbf{j}$ at the point $t = \sqrt{3}$.

3. If at a point on a curve $\mathbf{T} = <\frac{4}{5}, \frac{3}{5}>$, $\mathbf{a} = \ < 7, -1 >$, and $\mathbf{a_T} = 5 = \mathbf{a_N}$, compute $\mathbf{N}$.

4. Define carefully and precisely what it means for a function $f(x, y)$ to be continuous at a point (x_0, y_0).

5. Compute the indicated partial derivatives. (**Do not simplify**)

   (a) f_x where $f(x, y) = x^2 - 3y^2 + 7$

   (b) f_y where $f(x, y) = \ln\sqrt{xy}$

   (c) $\dfrac{\partial f}{\partial y \partial x}$ where $f(x, y) = x\sqrt{y}$

   (d) $\dfrac{\partial^2 f}{\partial y^2}$ where $f(x, y) = 3xy^2$

6. Compute the differential of $z = x^2 e^{3y}$.

7. Use the chain rule to find $\frac{dw}{dt}$ where $w = xy^2$, $x = e^t$, $y = \cos t$.

8. Find the directional derivative of $f(x, y) = x^2 + y^2$ at $(1, 1)$ in the direction $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$.

9. At the point $(9, 2)$ find a vector in the direction of maximum increase for the function $f(x, y) = e^y \sqrt{x}$.

10. Find an equation for the tangent plane to the surface $z = 3x^2 + y^2$ at the point $(-1, 2, 7)$.

11. Suppose that a function $f(x, y)$ satisfies

$$f_{xx} = 2, f_{xy} = f_{yx} = 1, f_{yy} = 3$$

at the critical point (a, b). What kind of extremum occurs at (a, b)? What if $f_{xx} = -2$?