Questions??
Trigonometry 6
VIII. New trig functions defined
   A. cosecant
   B. secant
   C. cotangent
   D. representation by line segments
   E. extended trig chart
      1. assignment 6.3

IX. Pythagorean Identities (3)
   A. get from unit circle and segment representations
      1. \( \sin^2 x + \cos^2 x = 1 \)
         demonstration with special angle
         demonstrate with calculator
         do x,y,r proof
      2. \( \tan^2 x + 1 = \sec^2 x \)
         demonstration with special angle
         demonstrate with calculator
         do x,y,r proof
      3. \( \cot^2 x + 1 = \csc^2 x \)
         demonstration with special angle
         demonstrate with calculator
         do x,y,r proof
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<th>π/3</th>
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\[ L^2 + 1^2 = h^2 \]
\[ 2^2 + 1^2 = 2^2 \]
\[ L^2 + 1^2 = 4 \]
\[ L^2 = 3 \]
\[ L = \sqrt{3} \]
Reciprocal Functions

\[
\frac{1}{\sin \theta} = \csc \theta \quad \frac{1}{\csc \theta} = \sin \theta
\]

\[
\frac{1}{\cos \theta} = \sec \theta \quad \frac{1}{\sec \theta} = \cos \theta
\]

\[
\frac{1}{\tan \theta} = \cot \theta \quad \frac{1}{\cot \theta} = \tan \theta
\]
\[
\begin{align*}
\cos^2 \theta + \sin^2 \theta &= 1 \\
(\frac{1}{2})^2 + \left(\frac{\sqrt{3}}{2}\right)^2 &= 1 \\
\frac{1}{4} + \frac{3}{4} &= 1
\end{align*}
\]
\[
\cos^2 \theta + \sin^2 \theta = 1
\]
\[
\left(\frac{x}{1}\right)^2 + \left(\frac{y}{1}\right)^2 = 1
\]
\[
x^2 + y^2 = 1
\]
\[
\tan^2 \theta + 1 = \sec^2 \theta
\]
\[
\left(\frac{y}{x}\right)^2 + 1 = \left(\frac{1}{x}\right)^2
\]
\[
\cot^2 \theta + 1 = \csc^2 \theta
\]
I. Roots in Geometry

A. Definition of Triangle Congruence

\[ \triangle ABC \cong \triangle DEF \]

1. \( \angle A = \angle D, \angle B = \angle E, \angle C = \angle F \)
2. \( AB = DE, BC = EF, AC = DF \)

B. Definition of Triangle Similarity

\[ \triangle ABC \sim \triangle DEF \]

1. \( \angle A = \angle D, \angle B = \angle E, \angle C = \angle F \)
2. \( \frac{AB}{AC} = \frac{BC}{DE} = \frac{AC}{DF} \)

C. Legal Ways to rewrite proportions

1. Starting Proportion: \( \frac{AB}{DE} = \frac{AC}{DF} \)
2. Alternation: \( \frac{AB}{AC} = \frac{DE}{DF} \)
3. Inversion: \( \frac{AB}{DE} = \frac{AC}{DF} \)
4. Addition: \( \frac{AB + DE}{DE} = \frac{AC + DF}{DF} \)
5. Subtraction: \( \frac{AB - DE}{DE} = \frac{AC - DF}{DF} \)
6. Any combinations of the above any number of times
7. Result of similarity: corresponding sides are proportional
8. This is what makes trigonometry possible
9. This is why the sine of 30 degrees is always 0.5

I. Roots in Geometry
   A. Definition of Triangle Congruence
      \[ \triangle ABC \cong \triangle DEF \]
      1. \( \angle A = \angle D, \angle B = \angle E, \angle C = \angle F \)
      2. \( AB = DE, BC = EF, AC = DF \)
   B. Definition of Triangle Similarity
      \[ \triangle ABC \sim \triangle DEF \]
      1. \( \angle A = \angle D, \angle B = \angle E, \angle C = \angle F \)
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   C. Legal Ways to rewrite proportions
      1. Starting Proportion: \( \frac{AB}{DE} = \frac{AC}{DF} \)
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      3. Inversion: \( \frac{AB}{AC} = \frac{DE}{DF} \)
      4. Addition: \( \frac{AB + DE}{AC + DF} \)
      5. Subtraction: \( \frac{AB - DE}{AC - DF} \)
      6. Any combinations of the above any number of times
      7. Result of similarity: corresponding sides are proportional
      8. This is what makes trigonometry possible
      9. This is why the sine of 30 degrees is always \( \frac{1}{2} \)

II. Standard Angle formation
   A. Origin is center of a circle with central angles as our angles
   B. Initial side always on positive x-axis
   C. Terminal side is \( x \) degrees in counter-clockwise motion from initial side
   D. Positive and negative angles
   E. Greek letters used for angles
III. Angle measure in radians
   A. Why? angle measure tied to size (radius) of circle.
   B. What is a radian?
   C. How many radians are there in 360 degrees?
   D. Radians in terms of pi
   E. Conversion from radians to degrees
   F. Conversion from degrees to radians
   G. Mapping out the circle in radians
      Assignment: 6.1

IV. Definition of 3 basic trig functions
   A. Sine
   B. Cosine
   C. Tangent
   D. SOHCAHTOA

V. Special angles
   A. degrees (Pi/6)
   B. degrees (Pi/4)
   C. degrees (Pi/3)
   D. Quadrantal angles (0, 90, 180, 270, 360, ...) [0, Pi/2, Pi, 3Pi/2, 2Pi, ...]

VI. Two special triangles
   A. 30-60-90 right triangle
      1. geometry theorem: the side opposite the 30 degree angle
         in a right triangle is one half the hypotenuse
   B. right triangle (isosceles Right Triangle)
      1. Use above theorems and the Pythagorean Theorem to
devlop ratios for all sides
         a) assignment 6.2

VII. Unit Circle (circle whose radius is 1)
   A. "Behavior" Charts
      1. line segments which represent the sine, cosine and
tangent
      2. coordinates of point on unit circle (x,y) are (cosine, sine)
      3. signs at quadrantal angles and in each quadrant
      4. incircle in each quadrant
      5. trig chart including
         0, 30, 45, 60, 90, 120, 135, 150, 180, 210, 225, 240, 270,
      6. negative angles
7. Angles over 360 degrees
   a) Assignment 6.3
8. Using a calculator
   a) Radians/degrees
   b) For most problems, I require exact answers
9. Using interpolation with tables

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E. Extended trig chart
   1. Assignment 6.3

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B. Derivative identity forms from the Pythagorean Identities
C. Do all functions in terms of sine, cosine, tangent, cosecant, secant, cotangent
   Assignment 7.1

X. Reciprocal Identities (Review) (3)
A. \( 1/\sin x = \cosecant \)
B. \( 1/\cos x = \secant \)
C. \( 1/\tan x = \cotangent \)

XI. Ratio Identities (2)
A. Tangent = \( \sin x/\cos x \)
B. Cotangent = \( \cos x/\sin x \)

More Identities
A. Sum and Difference (6)
B. Double Angle, Half Angle (6)
   Assignments 7.1, 7.2, 7.3

XII. Graphs of Sine and Cosine Functions
A. Amplitude
B. Period
C. shift
D. sum curves
   1. trig graph + constant	x
   2. trig graph + linear graph	x
   3. trig graph + trig	x

XIII. Other Trig Graphs
A. Tangent
B. Cosecant
C. Secant
D. Cotangent
   assignment 5.3, 5.4

Law of Sines, Cosines
Assignment 6.4, 6.5

Trigonometric Equations
Assignment 7.5

Trigonometric Forms of Complex Numbers
DeMoivre’s Theorem

Good Link: http://www.wlc.edu/academics/mat/calculus/index.aspx?id=1781